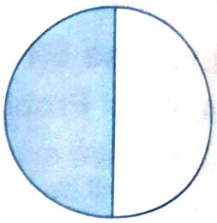
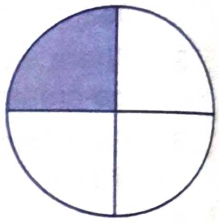
6 FRACTIONS

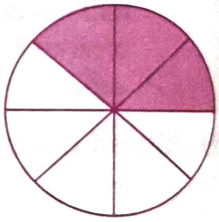
6.1 REVISION



In the adjacent figure, a circle is divided into 2 equal parts where one part is shaded. The shaded part is **one half** of the whole circle. We express one half by the symbol $\frac{1}{2}$. We read it as **one by two** or **one over two**. The unshaded part is also $\frac{1}{2}$ of the circle.

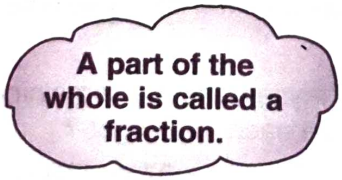


Here, the circle is divided into 4 equal parts in which one part is shaded. The shaded portion is one fourth ($\frac{1}{4}$) of the whole circle. We read $\frac{1}{4}$ as one by four or one over four.



Here, the circle is divided into 8 equal parts. Three parts are shaded. The shaded portion is $\frac{3}{8}$ of the whole circle. We read $\frac{3}{8}$ as three by eight or three over eight.

The numbers such as one half, one-third, one-fourth, two-third, three-fourth, one-sixth, etc., are called **fractional numbers** and their symbols $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{6}$, etc., are called **fractions**.



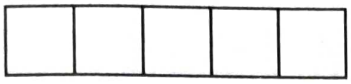
Numerator and Denominator of a Fraction.

We have already learnt that a fraction is written with two numerals arranged one over the other and are separated by a line '——'. The number under the line shows in how many equal parts the whole has been divided into. It is called **Denominator** of the fraction. The number above the line shows how many parts of the whole have been taken. It is called **Numerator** of the fraction. So in $\frac{3}{5}$, 3 is the numerator and 5 is the denominator.

Exercise 6(A)

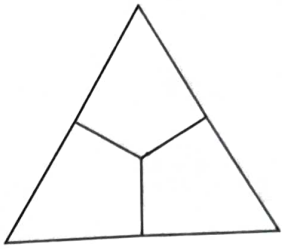
A. Colour the figures to match the fractions given.

1



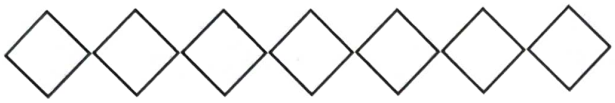
$\frac{3}{5}$

2



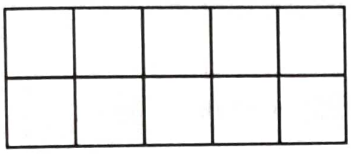
$\frac{2}{3}$

3



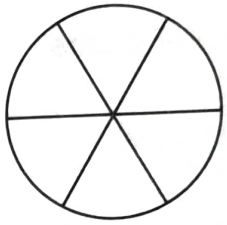
$\frac{5}{7}$

4



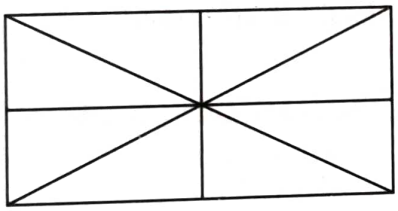
$\frac{7}{10}$

5



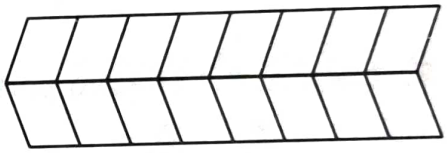
$\frac{3}{6}$

6



$\frac{3}{8}$

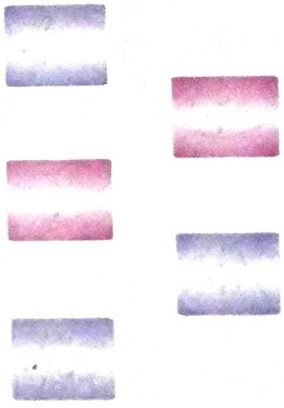
7



$\frac{9}{16}$

B. Write the fraction whose numerators and denominators are given below :

	Numerator	Denominator	Fraction
1	3	7	
2	6	11	
3	3	4	
4	2	9	
5	11	14	



C. Write the following fractions in words.

1 $\frac{3}{5}$

2 $\frac{4}{5}$

3 $\frac{8}{11}$

4 $\frac{9}{13}$

5 $\frac{7}{15}$

6 $\frac{5}{12}$

7 $\frac{6}{10}$

8 $\frac{4}{9}$

D. Write the fraction for each of the following :

1 Three-Fourths

2 Seven-nineths

3 Two-fifths

4 Eleven-fifteenths

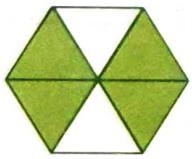
5 Three-sevenths

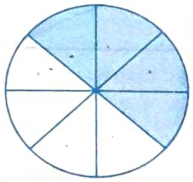
6 Seven-elevenths

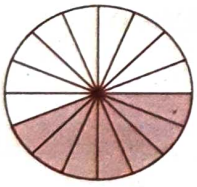
7 Nine-fourteenths

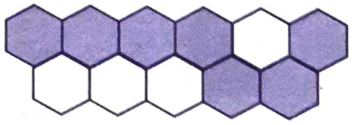
8 Eight-thirteenths

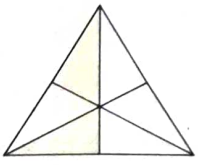
E. Look at the shaded parts in each figure. Write the fraction in the blanks.

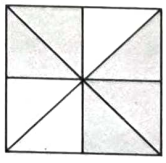
1  Fraction = ____

2  Fraction = ____

3  Fraction = ____

4  Fraction = ____

5  Fraction = ____

6  Fraction = ____

Properties of Fractions

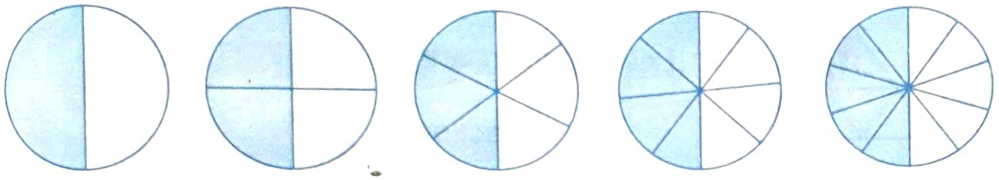
1. Any number multiplied or divided by 1 gives the number itself, the fraction does not change.

EXAMPLES $\frac{1}{2} \times 1 = \frac{1}{2}$, $\frac{4}{9} \times 1 = \frac{4}{9}$, $\frac{2}{5} \div 1 = \frac{2}{5}$, $\frac{3}{7} \div 1 = \frac{3}{7}$.

2. If we **multiply** or **divide** the numerator and denominator of a fraction by the same number (except zero) the value of the fraction does not change.

EXAMPLES $\frac{1}{2} \div \frac{1}{1} = \frac{1}{2}$, $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4} = \frac{1}{2}$, $\frac{1}{2} \times \frac{4}{4} = \frac{4}{8} = \frac{1}{2}$, $\frac{4}{8} \div \frac{4}{4} = \frac{1}{2}$, $\frac{8}{16} \div \frac{8}{8} = \frac{1}{2}$

6.2 EQUIVALENT FRACTIONS



$\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$ $\frac{5}{10}$

Equivalent fractions are two or more fractions which represent the same part of the whole.

All the above circles are of the same size. The shaded portions in the circles are also equal. Hence we can say that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$

Such fractions are called **Equivalent Fractions**. Also, in equivalent fractions the numerator of the first fraction multiplied by the denominator of the second fraction and the denominator of the first fraction multiplied by the numerator of the second fraction are equal.

EXAMPLE 1 **Check whether the following fractions are equivalent or not.**

(i) $\frac{5}{7}$ **and** $\frac{15}{21}$ **(ii)** $\frac{3}{7}$ **and** $\frac{24}{56}$ **(iii)** $\frac{3}{8}$ **and** $\frac{8}{12}$ **(iv)** $\frac{1}{6}$ **and** $\frac{4}{18}$.

(i) $\frac{5}{7} \times \frac{15}{21}$ OR $5 \times 21 = 105$, $15 \times 7 = 105$

(ii) $\frac{3}{7} \times \frac{24}{56}$ OR $3 \times 56 = 168$, $7 \times 24 = 168$

Since the products are equal hence, the fraction are equivalent.

(iii) $\frac{3}{8}$ and $\frac{8}{12}$ are equivalent if 3×12 and 8×8 are equal.

$\frac{3}{8} \times \frac{8}{12}$ OR $\begin{array}{rcl} 3 \times 12 & = & 36 \\ 8 \times 8 & = & 64 \end{array}$

Since, the result of 3×12 is not equal to 8×8. So, $\frac{3}{8}$ and $\frac{8}{12}$ are not equivalent fraction.

(iv) Similarly $\frac{1}{6}$ and $\frac{4}{18}$ are not equivalent fraction, as 1×18 and 6×4 are not equal.

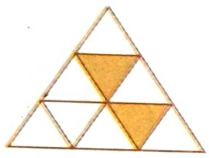
$\frac{1}{6} \times \frac{4}{18}$ OR $\begin{array}{rcl} 1 \times 18 & = & 18 \\ 6 \times 4 & = & 24 \end{array}$

EXAMPLE 2 **Look at the shaded parts in each figure. Write the fraction in the blanks. How will you prove that the fractions are equivalent ?**

Total parts = _9_

Shaded parts = _2_

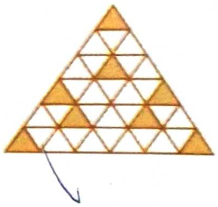
Fraction = $\frac{2}{9}$



Total parts = _36_

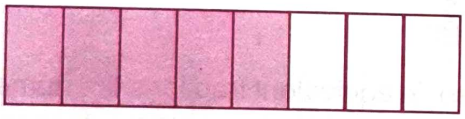
Shaded parts = _8_

Fraction = $\frac{8}{36}$



$2 \times 36 = 72$ and $8 \times 9 = 72$, so, $\frac{2}{9}$ and $\frac{8}{36}$ are equivalent fractions

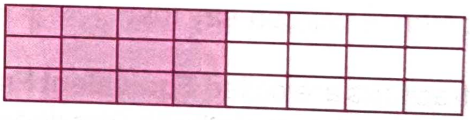
EXAMPLE 3 **Prove that $\frac{5}{8}$ is not equivalent to $\frac{12}{24}$.**



Total parts = _8_

Shaded parts = _5_

Fraction = $\frac{5}{8}$



Total parts = _24_

Shaded parts = _12_

Fraction = $\frac{12}{24}$

$5 \times 24 = 120$ and $8 \times 12 = 96$, so $\frac{5}{8}$ is not equivalent to $\frac{12}{24}$

EXAMPLE 4 **Write 4 fractions equivalent to the fraction $\frac{3}{4}$.**

The fractions equivalent to the fraction $\frac{3}{4}$ are

$\frac{3\times1}{4\times1}, \frac{3\times2}{4\times2}, \frac{3\times3}{4\times3}, \frac{3\times4}{4\times4}$ that is, $\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}$.

Note : By multiplying numerator and denominator by the same number. We can obtain any number of equivalent fractions.

Exercise 6(B)

A. Write three fractions equivalent to each of the following fractions.

1 $\frac{2}{5}$ —,—,— 2 $\frac{3}{7}$ —,—,— 3 $\frac{5}{8}$ —,—,— 4 $\frac{7}{10}$ —,—,—

B. Fill in the blanks so as to make equivalent fractions.

1 $\frac{5}{7} = \frac{20}{\square}$ 2 $\frac{12}{13} = \frac{\square}{39}$ 3 $\frac{5}{9} = \frac{\square}{45}$ 4 $\frac{6}{18} = \frac{\square}{9}$

5 $\frac{8}{12} = \frac{\square}{3}$ 6 $\frac{15}{25} = \frac{3}{\square}$ 7 $\frac{\square}{8} = \frac{20}{32}$ 8 $\frac{7}{\square} = \frac{28}{44}$

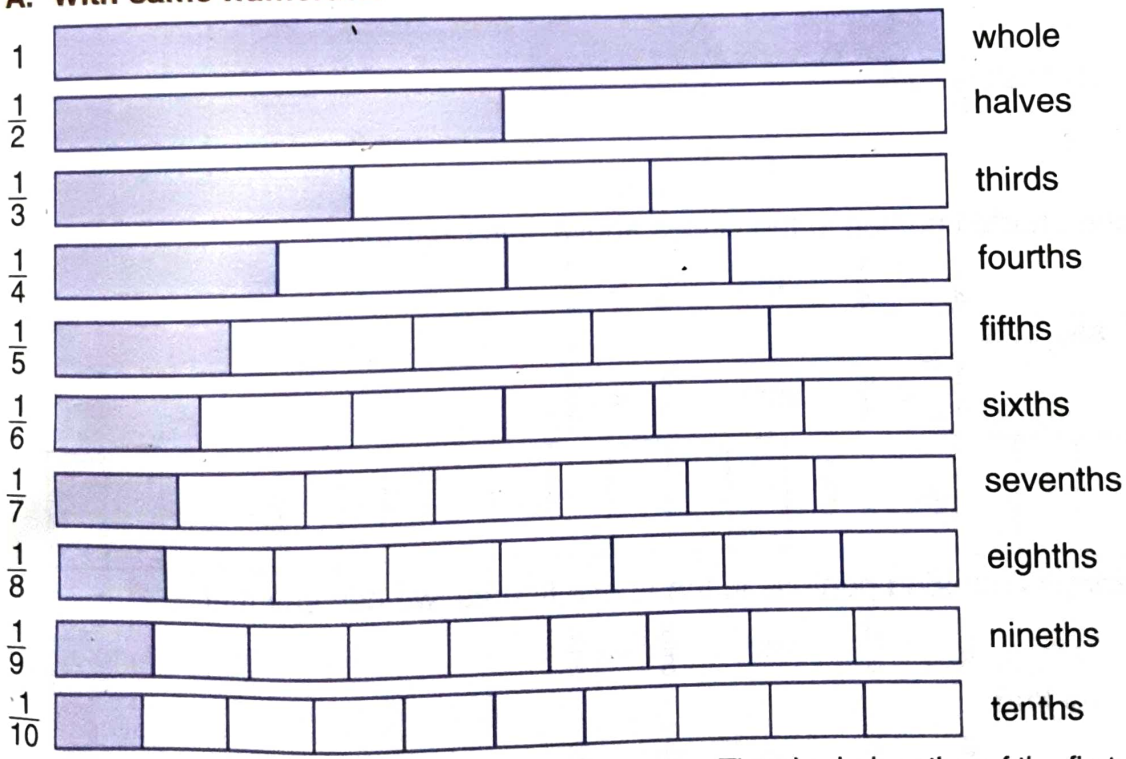
C. Which of the following pairs of fractions are equivalent fractions ?

1 $\frac{3}{4}$ and $\frac{12}{16}$ 2 $\frac{3}{4}$ and $\frac{15}{16}$ 3 $\frac{4}{12}$ and $\frac{5}{15}$ 4 $\frac{3}{5}$ and $\frac{12}{15}$

5 $\frac{6}{8}$ and $\frac{12}{21}$ 6 $\frac{9}{27}$ and $\frac{1}{3}$ 7 $\frac{5}{8}$ and $\frac{10}{12}$ 8 $\frac{2}{3}$ and $\frac{6}{9}$

6.3 ORDERING OF FRACTIONAL NUMBERS

A. With same numerator



Observe the shaded portions of these 10 similar strips. The shaded portion of the first strip (whole) is more than the shaded portion of the second (half portion), therefore, $1 > \frac{1}{2}$. The shaded portion of the second strip (half portion) is more than that of the third strip (one-third portion), therefore, $\frac{1}{2} > \frac{1}{3}$.

Similarly, we see that $\frac{1}{3} > \frac{1}{4}$, $\frac{1}{4} > \frac{1}{5}$, $\frac{1}{5} > \frac{1}{6}$, $\frac{1}{6} > \frac{1}{7}$, $\frac{1}{7} > \frac{1}{8}$, $\frac{1}{8} > \frac{1}{9}$ and $\frac{1}{9} > \frac{1}{10}$.

If we take any two strips say fourth and sixth, we find that $\frac{1}{4} > \frac{1}{6}$.

Similarly, from the shaded portions below we can say, $\frac{2}{3} > \frac{2}{4}$, $\frac{2}{4} > \frac{2}{5}$, $\frac{2}{5} > \frac{2}{6}$.

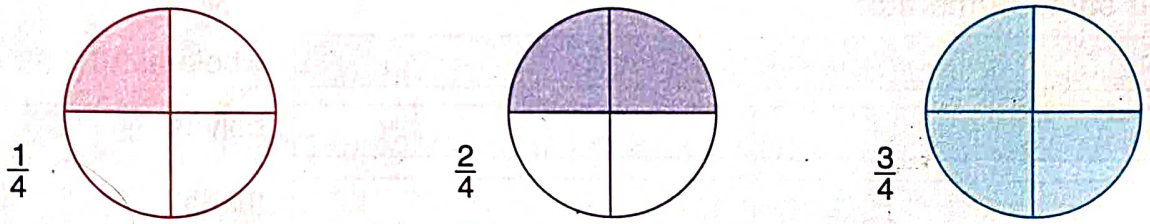
If two fractions have the **same numerator**, the fraction having **smaller denominator is greater** than the other.

EXAMPLE **Which is greater $\frac{3}{4}$ or $\frac{3}{5}$?**

Solution : $\frac{3}{4}$ and $\frac{3}{5}$ have the same numerator, but denominator 4 < denominator 5 ;

So, $\frac{3}{4} > \frac{3}{5}$.

B. With same denominator



Looking at, the shaded portion in the circles above, it is clear that

$\frac{2}{4} > \frac{1}{4}$ and $\frac{3}{4} > \frac{2}{4}$

We can also say that,

$\frac{1}{4} < \frac{2}{4}$ and $\frac{2}{4} < \frac{3}{4}$

Looking at, from the shaded portions in the above figures, we can say that

$\frac{2}{5} < \frac{3}{5}$ and $\frac{3}{5} < \frac{4}{5}$

We can also say that

$\frac{3}{5} > \frac{2}{5}$ and $\frac{4}{5} > \frac{3}{5}$

If two fractions have the **same denominator**, the fraction with the **greater numerator is greater** than the other.

EXAMPLE 1 **Which is greater $\frac{2}{5}$ or $\frac{4}{5}$?**

Both the fractions have the same denominator but (numerator) 4 > (numerator) 2.

So, $\frac{4}{5} > \frac{2}{5}$

EXAMPLE 2 **Which is smaller $\frac{5}{8}$ or $\frac{7}{8}$?**

Both the fractions have the same denominator but 7 > 5.

So, $\frac{5}{8} < \frac{7}{8}$

EXAMPLE 3 **Arrange the following fractions in descending order $\frac{2}{5}$, $\frac{4}{5}$, $\frac{3}{5}$, $\frac{1}{5}$.**

Fractions have the same denominator.

Numerators of the fractions are 4, 3, 2 and 1.

Numerator 4 > numerator 3 > numerator 2 > numerator 1.

$\therefore \frac{4}{5} > \frac{3}{5}, \frac{3}{5} > \frac{2}{5}, \frac{2}{5} > \frac{1}{5}$. Thus, the order is : $\frac{4}{5}, \frac{3}{5}, \frac{2}{5}, \frac{1}{5}$

EXAMPLE 4 **Arrange the following fractions in ascending order $\frac{4}{5}$, $\frac{4}{7}$, $\frac{4}{11}$, $\frac{4}{9}$.**

Fractions have the same numerator.

Denominators of the fractions are 5, 7, 11 and 9.

If two fractions have the **same numerator**, the fraction having **smaller denominator is greater** than the other.

As such $\frac{4}{5} > \frac{4}{7} > \frac{4}{9} > \frac{4}{11}$ Or, $\frac{4}{11} < \frac{4}{9}, \frac{4}{9} < \frac{4}{7}, \frac{4}{7} < \frac{4}{5}$

Thus the order is : $\frac{4}{11}, \frac{4}{9}, \frac{4}{7}, \frac{4}{5}$

Exercise 6(C)

A. Fill in the blanks by using ">" or "<" to make correct statements.

1	$\frac{3}{4}$ ▢ $\frac{1}{4}$	2	$\frac{6}{11}$ ▢ $\frac{6}{7}$	3	$\frac{8}{11}$ ▢ $\frac{9}{11}$
4	$\frac{4}{5}$ ▢ $\frac{2}{5}$	5	$\frac{8}{9}$ ▢ $\frac{8}{13}$	6	$\frac{3}{5}$ ▢ $\frac{3}{7}$
7	$\frac{2}{7}$ ▢ $\frac{5}{7}$	8	$\frac{3}{8}$ ▢ $\frac{3}{4}$	9	$\frac{11}{13}$ ▢ $\frac{9}{13}$
10	$\frac{10}{13}$ ▢ $\frac{12}{13}$	11	$\frac{4}{5}$ ▢ $\frac{4}{7}$	12	$\frac{9}{12}$ ▢ $\frac{7}{12}$

B. Circle the greater of the two given fractions.

1	$\frac{2}{5}$	$\frac{4}{5}$	2	$\frac{4}{7}$	$\frac{1}{7}$	3	$\frac{3}{4}$	$\frac{1}{4}$	4	$\frac{2}{6}$	$\frac{3}{6}$
5	$\frac{3}{8}$	$\frac{5}{8}$	6	$\frac{2}{3}$	$\frac{2}{5}$	7	$\frac{4}{5}$	$\frac{4}{9}$	8	$\frac{3}{7}$	$\frac{3}{5}$

C. Arrange the following fractions in ascending order.

1. $\frac{3}{5}, \frac{2}{5}, \frac{4}{5}$
2. $\frac{2}{7}, \frac{5}{7}, \frac{3}{7}$
3. $\frac{3}{8}, \frac{1}{8}, \frac{7}{8}$
4. $\frac{4}{5}, \frac{4}{9}, \frac{4}{8}, \frac{4}{6}$
5. $\frac{5}{9}, \frac{5}{7}, \frac{5}{6}, \frac{5}{8}$
6. $\frac{7}{11}, \frac{7}{13}, \frac{7}{9}$
7. $\frac{8}{11}, \frac{4}{11}, \frac{6}{11}, \frac{2}{11}$
8. $\frac{3}{6}, \frac{3}{8}, \frac{3}{5}$

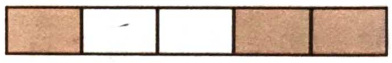
D. Arrange the following fractions in descending order.

1. $\frac{1}{4}, \frac{3}{4}, \frac{2}{4}$
2. $\frac{2}{5}, \frac{4}{5}, \frac{3}{5}$
3. $\frac{6}{7}, \frac{1}{7}, \frac{2}{7}$
4. $\frac{4}{7}, \frac{4}{5}, \frac{4}{9}, \frac{4}{6}$
5. $\frac{5}{8}, \frac{5}{6}, \frac{5}{9}, \frac{5}{7}$
6. $\frac{7}{10}, \frac{7}{13}, \frac{7}{11}, \frac{7}{8}$
7. $\frac{2}{6}, \frac{1}{6}, \frac{5}{6}, \frac{3}{6}$
8. $\frac{1}{11}, \frac{1}{6}, \frac{1}{9}$

6.4 ADDITION OF FRACTIONS

Here, a strip of paper is divided into 5 equal parts.

One part on the left and 2 parts on the right of the paper strip are shown shaded. Observe that 1 shaded part on the left represents $\frac{1}{5}$ of the whole and 2 shaded parts on the right represent $\frac{2}{5}$ of the whole.



We know that 1 shaded part on the left taken together with 2 shaded parts on the right will give 3 shaded parts.

Thus, $\frac{1}{5} + \frac{2}{5} = \frac{1+2}{5}$ or $\frac{3}{5}$.

Similarly, the shaded parts of the circle, given alongside, gives

$$\frac{1}{6} + \frac{3}{6} = \frac{1+3}{6} \text{ or } \frac{4}{6}$$

The shaded parts of the strip gives

$$\frac{2}{7} + \frac{1}{7} + \frac{2}{7} = \frac{2+1+2}{7} \text{ or } \frac{5}{7}$$

What do we observe?

We note that the sum of two or more fractions having the same denominator is

$$= \frac{\text{Sum of numerators}}{\text{Denominator}}$$

Exercise 6(D)

A. Add the following fractions.

1 $\frac{3}{13} + \frac{8}{13}$ 2 $\frac{4}{11} + \frac{3}{11} + \frac{2}{11}$ 3 $\frac{7}{19} + \frac{4}{19} + \frac{6}{19}$ 4 $\frac{4}{17} + \frac{2}{17} + \frac{7}{17}$

5 $\frac{7}{20} + \frac{9}{20} + \frac{1}{20}$ 6 $\frac{8}{25} + \frac{9}{25} + \frac{2}{25}$ 7 $\frac{5}{23} + \frac{8}{23} + \frac{10}{23}$ 8 $\frac{4}{21} + \frac{11}{21} + \frac{6}{21}$

B. Fill in the boxes to complete the equation.

1 $\frac{8}{29} + \frac{11}{29} = \frac{\square}{29}$ 2 $\frac{11}{31} + \frac{14}{31} = \frac{\square}{31}$

3 $\frac{7}{20} + \frac{\square}{20} = \frac{9}{20}$ 4 $\frac{\square}{23} + \frac{11}{23} = \frac{19}{23}$

5 $\frac{\square}{17} + \frac{7}{17} = \frac{10}{17}$ 6 $\frac{5}{19} + \frac{\square}{19} = \frac{11}{19}$

7 $\frac{9}{25} + \frac{\square}{25} = \frac{14}{25}$ 8 $\frac{13}{29} + \frac{5}{29} = \frac{\square}{\square}$

6.5 SUBTRACTION OF FRACTIONS

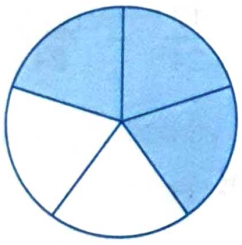


Fig.1

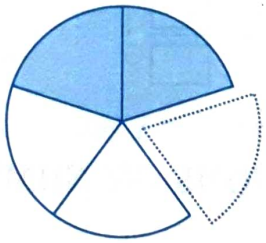


Fig. 2

In Fig. 1, circle is divided into 5 equal parts. 3 parts are shaded. The shaded portions represent the fraction $\frac{3}{5}$.

In Fig. 2, one shaded part is removed which represents the fraction $\frac{1}{5}$, we are left with only 2 parts of the shaded portion which represents $\frac{2}{5}$.

$$\frac{3}{5} - \frac{1}{5} = \frac{2}{5}$$

So, we note that subtraction of two or more fractions having the same denominator is

$$= \frac{\text{Difference in numerators}}{\text{Denominator}}$$

Exercise 6(E)

A. Subtract the following.

1 $\frac{14}{19} - \frac{7}{19}$

2 $\frac{14}{23} - \frac{8}{23}$

3 $\frac{15}{17} - \frac{9}{17}$

4 $\frac{19}{31} - \frac{11}{31}$

5 $\frac{23}{29} - \frac{18}{29}$

6 $\frac{27}{42} - \frac{18}{42}$

7 $\frac{26}{37} - \frac{19}{37}$

8 $\frac{31}{43} - \frac{19}{43}$

B. Fill in the boxes to complete the equations.

1 $\frac{4}{9} - \frac{2}{9} = \frac{\square}{9}$

2 $\frac{8}{11} - \frac{5}{11} = \frac{\square}{11}$

3 $\frac{7}{13} - \frac{5}{13} = \frac{2}{\square}$

4 $\frac{\square}{15} - \frac{4}{15} = \frac{8}{15}$

5 $\frac{15}{18} - \frac{\square}{18} = \frac{11}{18}$

6 $\frac{\square}{12} - \frac{4}{12} = \frac{6}{12}$

7 $\frac{\square}{16} - \frac{9}{16} = \frac{5}{16}$

8 $\frac{8}{14} - \frac{7}{14} = \frac{\square}{\square}$

9 $\frac{11}{23} - \frac{8}{23} = \frac{\square}{\square}$

10 $\frac{17}{24} - \frac{\square}{24} = \frac{7}{24}$